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BI-MODAL FATIGUE CURVES (U)  
MAY 80 R G LAMBERT

N00019-78-C-0407  
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6 **BI-MODAL FATIGUE CURVES**

10 R.G. LAMBERT

15 THIS WORK WAS PERFORMED ON NAVY CONTRACT/N00019-78C-0497

11 MAY 1980

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AD A103210

GENERAL ELECTRIC COMPANY  
AEROSPACE ELECTRONIC SYSTEMS DEPARTMENT  
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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO. <i>AD-A103210</i>	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle)  BI-MODAL Fatigue Curves		5. TYPE OF REPORT & PERIOD COVERED
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s)  R. G. Lambert		8. CONTRACT OR GRANT NUMBER(s)  Navy Contract N00019-78-C-0407
9. PERFORMING ORGANIZATION NAME AND ADDRESS General Electric Company Aerospace Electronic Systems Division Utica, New York 13503		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Electronic Systems Command Washington, D.C. 20360 Attn: Code 8131		12. REPORT DATE May 1980
		13. NUMBER OF PAGES 21
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report)  Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report)  Distribution Unlimited. Approved for Public Release.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number)		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number)		

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# BI-MODAL FATIGUE CURVES

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## INTRODUCTION

↓  
The objective of screening tests is to detect workmanship defects in electronic "black boxes" prior to delivery to the customer. Such tests typically involve subjecting the units under test (UUT) to temperature cycling and random vibration tests. All of the structural elements within the UUT (e.g. the individual piece parts, solder joints, leads wires, support structures) cumulate fatigue damage during the screening test and subsequently during the intended service environment. The cumulative fatigue damage may range from very little to very large depending upon the individual stress levels and number of stress cycles experienced by each structural element.

The desire is to have the defective elements fail (i.e. fracture) and be replaced without cumulating significant damage to the non-defective elements. This desire assumes that each structural element type fatigue curve is bi-modal (i.e. has two peaks) and that the corresponding groupings of defective and non-defective elements are disjoint as shown in figure 1.

↓  
This study will develop the approach to predicting fatigue life and mechanical reliability for such structural elements. The results of this study would be used to analyze the mechanical reliability of all of the structural elements in the UUT as a whole.

↖

## FATIGUE CURVE REPRESENTATION

References [1] and [2] show the fatigue curve representations for sinusoidal and random stress situations that cover both the low and high cycle fatigue regions.

For sinusoidal stresses

$$S = \bar{A} N_s^{-1/\beta} \quad (1)$$

where  $S$  = sinusoidal stress (ksi)  
 $\bar{A}$  = ultimate cyclic true strength (ksi)  
 $\beta$  = slope parameter  
 $N_s$  = number of stress cycles to failure

For random stresses

$$\sigma = \bar{C} N_m^{-1/\beta} \quad (2)$$

where  $\sigma$  = rms stress level (ksi)  
 $\bar{C}$  = strength parameter (ksi)  
 $\beta$  = slope parameter  
 $N_m$  = median cycles to failure

$$\bar{C} = \left[ \frac{\bar{A}}{\sqrt{2}} \right] \left[ \frac{1}{\Gamma\left(\frac{2+\beta}{2}\right)} \right]^{1/\beta} \quad (3)$$

For low cycle fatigue situations

$$\epsilon = \bar{\epsilon}_u N_f^{-1/2} \quad (4)$$

where  $\epsilon$  = applied cyclic strain (in/in)  
 $\bar{\epsilon}_u$  = material ductility parameter (in/in)  
 $N_f$  = median cycles to failure

Fatigue life, failure probability and cumulated fatigue damage are functions of the cyclic fatigue curve parameters  $\bar{A}$ ,  $\bar{\epsilon}_u$  and  $\beta$ .  $\bar{A}$  represents the fatigue strength (ksi) which is important in the elastic high cycle fatigue region.  $\bar{\epsilon}_u$  represents the material's ductility which is important in the inelastic (plastic) low cycle fatigue region. Both  $\bar{A}$  and  $\bar{\epsilon}_u$  can be thought of as "y-intercepts" of the fatigue curves on a log-log plot.  $\beta$  is a slope parameter.  $\beta \approx 2$  for most structural materials in the low cycle fatigue region.  $\beta \approx 9$  for ductile materials and  $\beta \approx 20$  for brittle materials in the high cycle fatigue region. Initial cracks reduce  $\beta$  by a factor of approximately 2.

Fatigue life, failure probability and cumulated fatigue damage are also functions of applied stresses and strains and their corresponding variances.

The statements in the two previous paragraphs are equally valid for both sinusoidal and random applied stress or strains.



## ELEMENT GROUPING

Figure 1 shows the grouping of the defective and non-defective parts for a given structural element type.  $\bar{A}$  is the general fatigue curve strength parameter.  $p(\bar{A})$  is the probability density function of  $\bar{A}$ . If a random stress situation is being considered,  $\bar{C}$  should be used instead of  $\bar{A}$ . If a low cycle fatigue situation is being considered,  $\bar{\epsilon}_u$  should be used instead of  $\bar{A}$ . For simplicity this study will hereafter refer only to  $\bar{A}$ .

Group 1 represents the lower strength of the defective parts of an element type. Group 2 represents the higher strength of the non-defective parts.  $\Delta_1$  and  $\Delta_2$  represent the standard deviation of the respective group fatigue curves. The two groupings collectively represent the single bi-modal fatigue curve of the structural element type.

Descriptive words to distinguish between the two groups other than defective/non-defective would be weak/strong, abnormal/normal respectively. Group 1 is systematically different from Group 2. The difference may occur gradually or suddenly as a function of time. The two groups may even represent the same part type from two different manufacturers. As an example Group 1 might represent solder joint strength from a new or improperly trained operator whereas Group 2 would represent the strength from an experienced or properly trained operator.  $\Delta_2$  represents normal variances in the manufacturing process (e.g.  $\Delta_2$  may be large for a particular non-automated process).

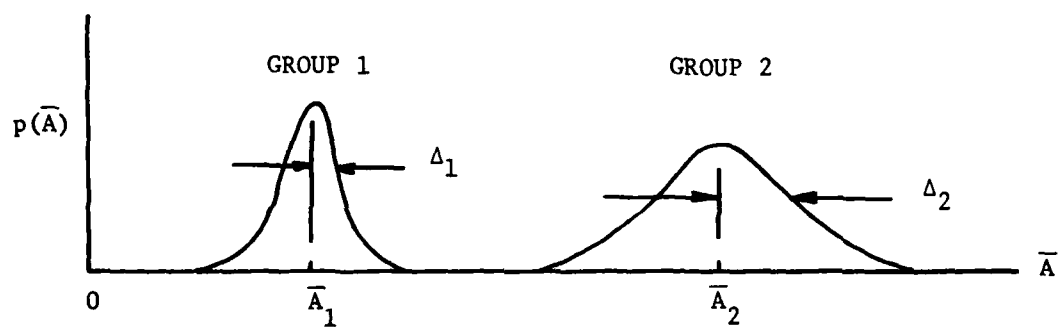


FIGURE 1  
BI-MODAL STRENGTH PARAMETER DISTRIBUTION

## TYPICAL DEFECT EXAMPLES

Several examples of defects that reduce fatigue strength are as follows: improper annealing of a major structural element, improper blend of Tin and Lead in solder, improper temper in a dip brazed aluminum chassis, improper surface plating or surface treatment, over shoot-peening a surface, impurities, fretting and corroding environments. Some of the above defects alter both  $\bar{A}$  and  $\beta$ , some alter only  $\bar{A}$ . See Appendix A [3] and references [4] [5].  $\bar{A}$  and  $\beta$  are both different between the two curves of figures 4 and 6. Only  $\bar{A}$  is different between the two curves of figures 5 and 7. Several examples of defects that reduce the ductility of electrodeposited copper for low cycle fatigue applications are as follows: improper chemical composition, plating current density, plating temperature and impurities of the plating bath. [6] [7]

## DEFECT CLASSES

Close scrutiny of various defect types leads to the statement that all defects do not create bi-modal fatigue curves as previously discussed. Some do. Others do not. For example, notches inadvertently put into a structural element would result in an increase in applied stress locally at the notch; not a decrease in strength. This is discussed further in the section on SHARP NOTCHES. A similar example would be for a multilayer board (MLB) that was improperly cured. The coefficient of thermal expansion in the direction perpendicular to the plane of the MLB would be abnormally high, thereby causing an unusually large applied strain; not a decrease in ductility of the electrodeposited copper plated-through-holes.

Still another example of a defect causing a stress increase is a lead wire from, say, a connector to an MLB with no stress relief. As the MLB vibrates

as its resonant frequency the stress in the taught wire is unusually large compared to what it would be with slack in the wire.

A third general class of defects would be the class where initial cracks are present. Such cases are the basis of the discipline of Fracture Mechanics. An example would be an improperly adjusted bonding machine for the internal fabrication of microelectronic device whereby the bonding tool came down too far and cracked (but not fractured) the lead wire that it was attempting to bond to a pad.

In summary defects fall into three classes: (1) those that reduce fatigue strength or ductility, (2) those that cause increases in the applied stress or strain and (3) those that introduce initial cracks.

## SHARP NOTCHES

Sharp notches can be equivalently thought of as either stress raisers or fatigue strength reducers. The modern methods of fatigue analysis would be to perform an elastic - plastic analysis of the local stress - strain at the notch using finite element methods or Neuber's method. Neuber's method is relatively simple to use and is accurate for steel but not always accurate for aluminum alloys. A fatigue strength reduction factor is calculated for the notch. The finite element method gives values to the high local stresses and strains. These values can be directly used to predict fatigue life by comparing them with un-notched specimen fatigue strength data. That is, the fatigue strength is not reduced. Rather the stress at the notch is increased. The finite element method can be accurately applied for all materials.

In this study the finite element method is recommended as the first choice. The value of the notch local stress/strain is inserted as an applied stress or strain in the computation of the median cycles to failure  $N_m$ . Correspondingly the increase in local stress over the nominal value should be included in the constant  $C_4$  (ksi/g rms) if the fatigue life is to be related to the electronic "black box" input acceleration level  $\ddot{x}$  (g rms) by the expression  $\sigma = C_4 \ddot{x}^\eta$ .  $\eta$  is the damping linearity term. Any variances in the local notch stresses or strains should be entered as  $\delta^2$  values into the variance term  $\psi^2$  which includes the effects of simultaneous stress or strain variances  $\delta^2$  and strength variances  $\Delta^2$  for predicting failure probabilities or cycles to first failure.

If Neuber's method is used, the value of  $\bar{A}$  or  $\bar{\epsilon}_u$  should be reduced by the calculated strength reduction factor  $K_f$ . Any variances in  $\bar{A}$  should be entered as  $\Delta^2$  values into the variance term  $\psi^2$ .

#### FRACTURE MECHANICS EFFECTS

As the sharpness of the notch increases (i.e. as the notch root radius decreases) there comes a point beyond which the typical "fatigue analysis" ends and "fracture mechanics analysis" begins.

Figures 2 and 3 illustrate the value of root radius below which the apparent fracture toughness  $K_{IC}$  remains constant. Fracture mechanics analysis should be used for cases where  $K_{IC}$  remains constant. Such analyses are described in other sections of the study and will not be repeated here. [8] [9] [10] and Attachment A.

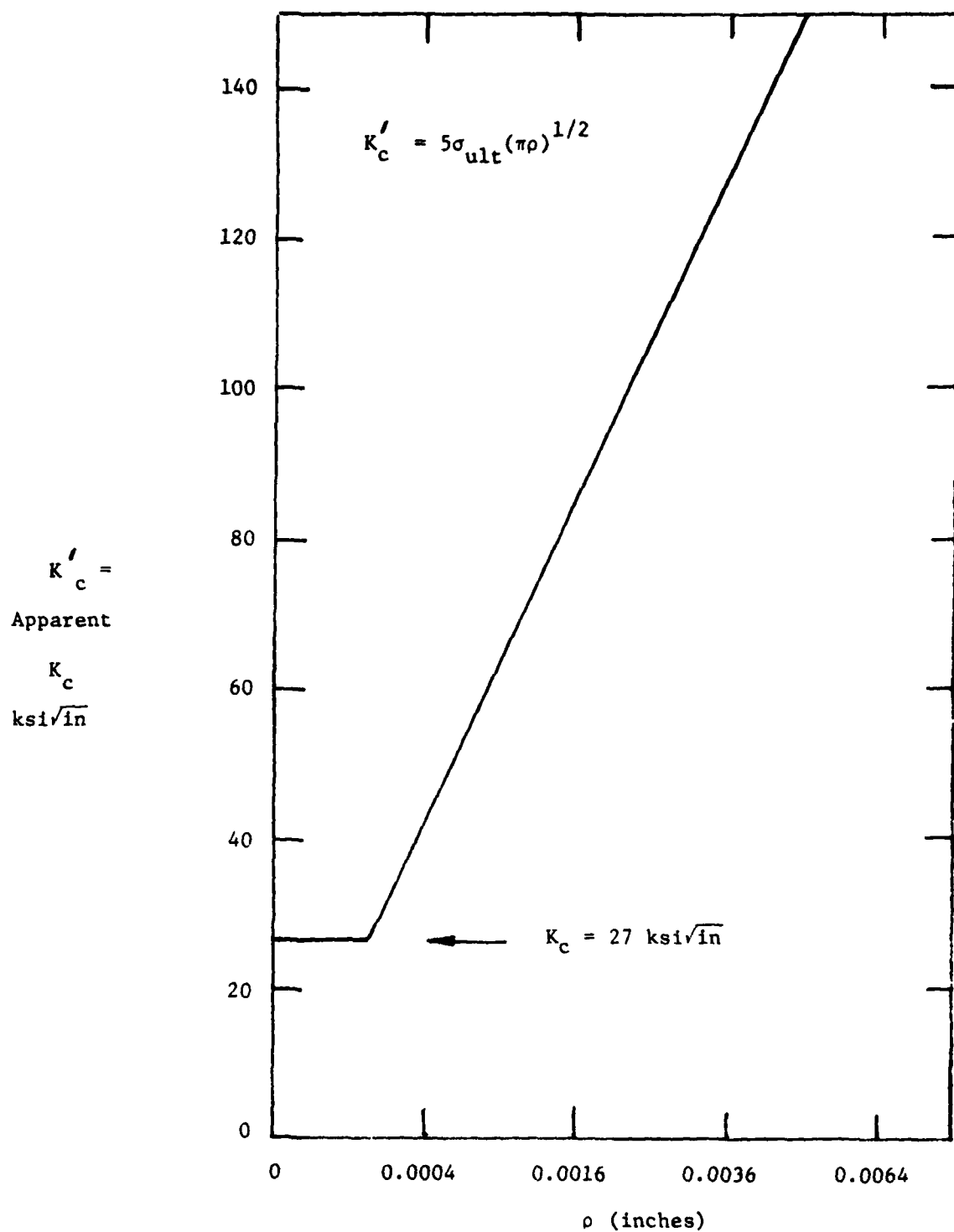


Figure 2 Influence of Root Radius on Fracture Toughness of H-11 Steel

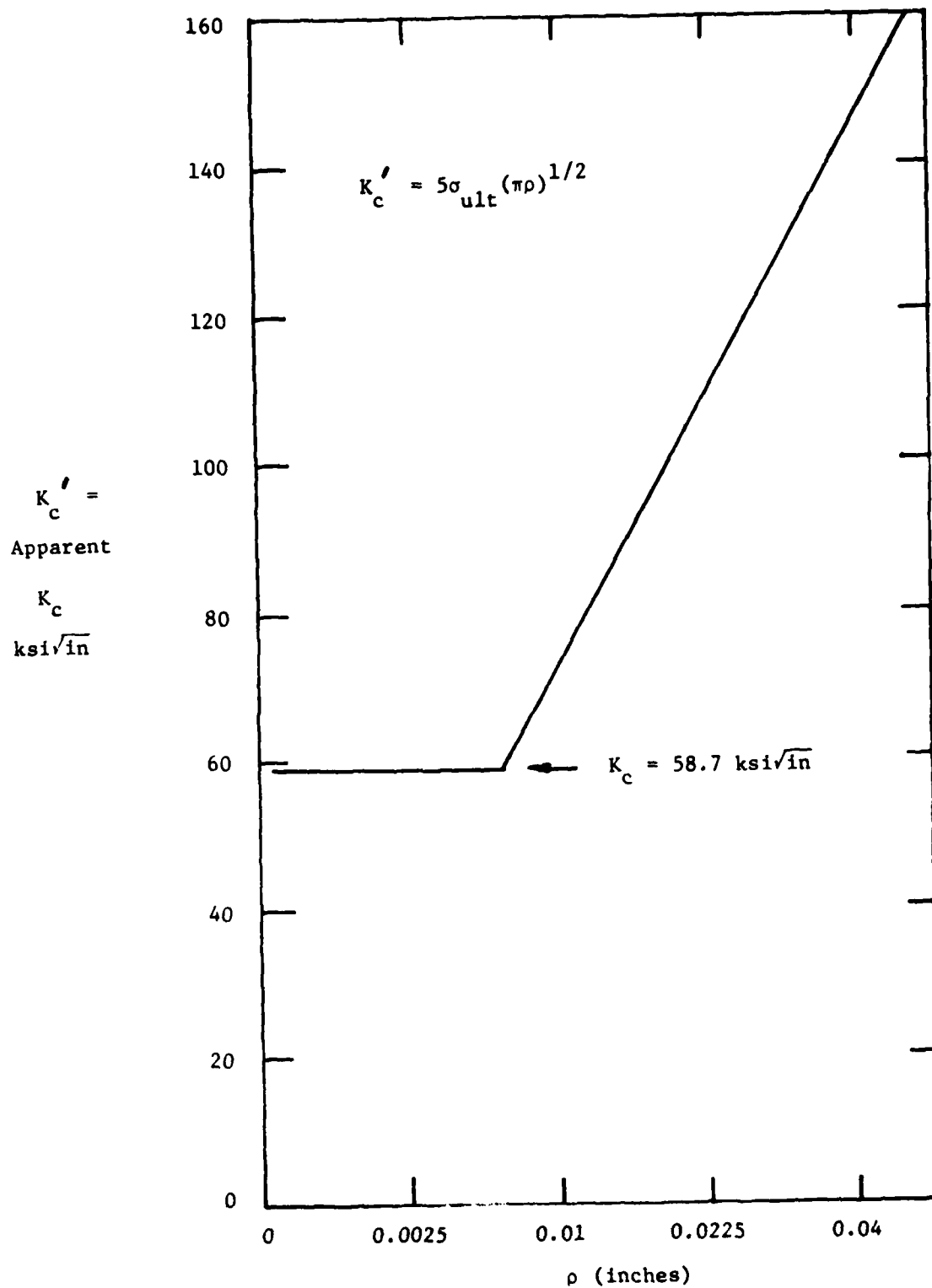


Figure 3 Influence of Root Radius on Fracture Toughness of Aluminum Alloy 7075-T6



## STATIC STRESSES

Tensile or compressive static stresses do not affect fatigue life in the low cycle fatigue region due to stress relaxation effects. However, they do affect fatigue life in the high cycle fatigue region.

Static stress can be residual stresses as a result of the manufacturing process (e.g. in castings, welded structures, nitriding or shot peening). They may result from the application of static loads. They may result from certain types of cyclic overloads.

The amount of reduction or enhancement is beyond the scope of this study. However, it will be stated here without proof that static stresses reduce or increase  $\bar{A}$  but do not affect  $\beta$ . Tensile static stresses decrease the value of  $\bar{A}$ . Compressive static stress increase the value of  $\bar{A}$ . Define the static stress as  $\sigma_0$ .  $\sigma_0$  is positive for tensile stresses and negative for compressive stresses. Quantitatively the effect of  $\sigma_0$  is to modify  $\bar{A}$  by subtracting  $(\bar{A} - \sigma_0)$  in place of  $\bar{A}$  in all of the fatigue life and reliability equations involving  $\bar{A}$ . [11]

## DISJOINTEDNESS

Some defects will show only subtle degradation of fatigue strength characteristics. As examples consider a part that is only slightly improperly annealed or a solder joint that is only slightly "cold". Groups 1 and 2 of figure 1 may not be very disjoint. In some cases the fatigue curve may show only a large standard deviation  $\Delta_2$  and not even be bi-modal. For those cases it will be difficult to devise a screening test level and duration that will detect the defect without cumulating more damage on the non-defective parts than desired.

## FAILURE PROBABILITIES

Appendix B shows the derivation of the appropriate failure probability expressions. Also shown are typical curves of  $F(N)$  versus  $N$  to be expected. Figure 9 shows that an appropriate screening test level and duration can be obtained to cause failure of only the defective elements. Group 1 and 2 disjointedness is a prerequisite.

## CUMULATED DAMAGE

The expressions of Appendix B can be used along with Attachment A methods to calculate the damage cumulated during the screening test and during the subsequent service environment. Group 1 and Group 2 elements should be analyzed separately for this purpose.

The cumulated damage of reworked or replaced defects during the service environment can also be calculated using the methods of Attachment A.

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APPENDIX A

FATIGUE CURVE EXAMPLES

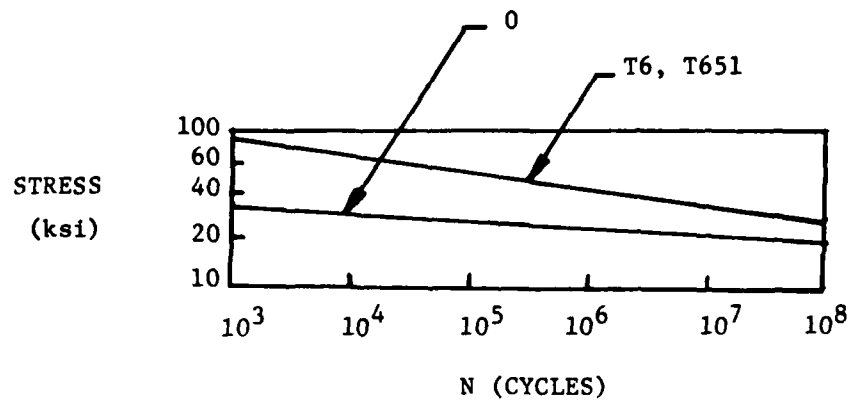


FIGURE 4 FATIGUE CURVE FOR 7075 ALUMINUM ALLOY - REVERSED BENDING

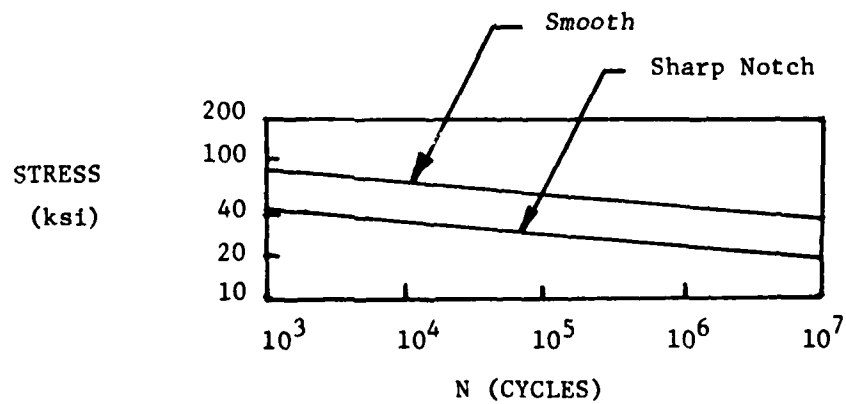


FIGURE 5 FATIGUE CURVE FOR TITANIUM ALLOY (Ti-8Al-1Mo-IV sheet)

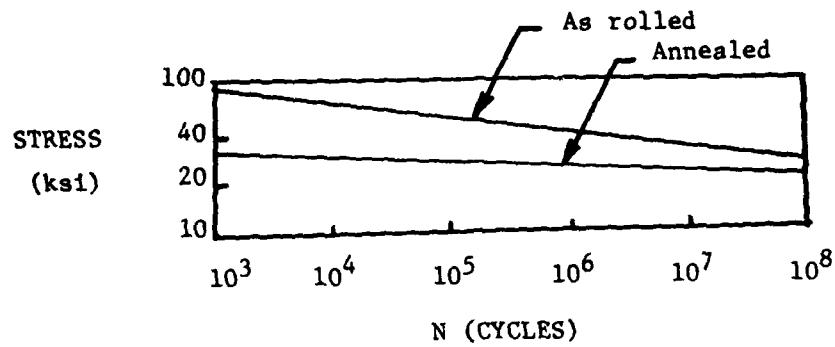


FIGURE 6 FATIGUE CURVE OF BRASS (60% COPPER - 40% ZINC)

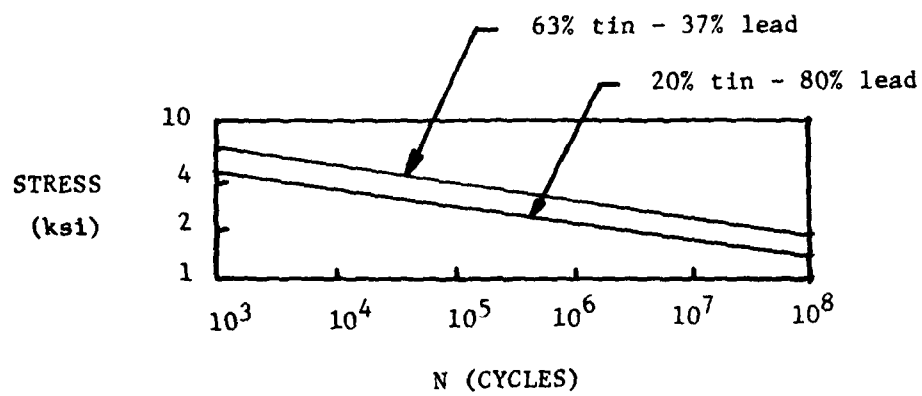


FIGURE 7 FATIGUE CURVE FOR SOFT SOLDER

APPENDIX B

FAILURE PROBABILITY DERIVATION

Refer to Figure 1.

$$\int_0^{\infty} p(A_1) dA = q_1 \quad (6)$$

$$\int_0^{\infty} p(A_2) dA = q_2 = 1 - q_1 \quad (7)$$

$$P(A + B) = P(A) + P(B) - P(AB)$$

Define  $P(A + B)$  as the probability of A or B failing where

$$A = \{A_1\} ; B = \{A_2\} .$$

If the system contains only one component of the class  $\{A_1, A_2\}$ , then

$$P(A_1 \oplus A_2) = P(A_1) + P(A_2)$$

In the above case  $A_1$  and  $A_2$  are mutually exclusive.

If the system is more complex and contains several components from the set  $\{A_1, A_2\}$ , then both  $A_1$  and  $A_2$  parts could fail simultaneously. Their failures would be independent generally.

Thus

$$P(A_1 + A_2) = P(A_1) + P(A_2) - P(A_1)P(A_2) \quad (8)$$

$$P(A_1) = F_1(N) = q_1 \left\{ 0.5 + \operatorname{erf}_p \left[ \frac{\bar{A}_1}{\Delta_1} \left\{ \left( \frac{N}{N_{m1}} \right)^{1/\beta_1} - 1 \right\} \right] \right\} \quad (9)$$



$$P(A_2) = F_2(N) = q_2 \left\{ 0.5 + \operatorname{erf}_p \left[ \frac{\bar{A}_2}{\Delta_2} \left\{ \left( \frac{N}{N_{m2}} \right)^{1/\beta_2} - 1 \right\} \right] \right\} \quad (10)$$

$$F(N) = F_1(N) + F_2(N) - [F_1(N)][F_2(N)] \quad (11)$$

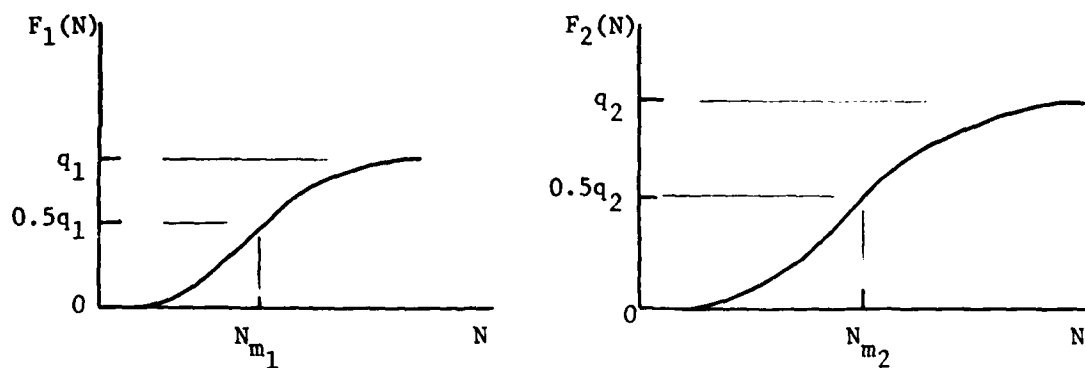


FIGURE 8 SUB-SET FAILURE PROBABILITIES

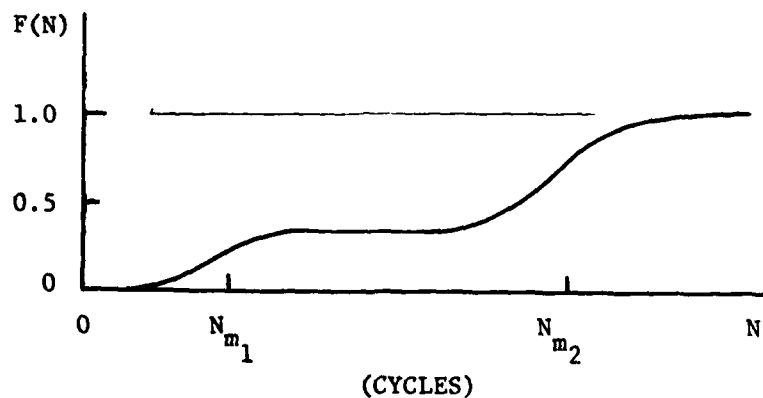


FIGURE 9 COMPOSITE FAILURE PROBABILITY

$$p(N) = p_1(N) + p_2(N) - p_1(N)p_2(N) \quad (12)$$

$$p_1(N) = \frac{q_1 \bar{A}_1}{\beta_1 \psi_1 \sqrt{2\pi} N} \left( \frac{N}{N_{m1}} \right)^{1/\beta_1} \exp \left[ - \frac{\bar{A}_1^2 \left\{ \left( \frac{N}{N_{m1}} \right)^{1/\beta_1} - 1 \right\}^2}{2\psi_1^2} \right] \quad (13)$$

$$p_2(N) = \frac{q_2 \bar{A}_2}{\beta_2 \psi_2 \sqrt{2\pi} N} \left( \frac{N}{N_{m2}} \right)^{1/\beta_2} \exp \left[ - \frac{\bar{A}_2^2 \left\{ \left( \frac{N}{N_{m2}} \right)^{1/\beta_2} - 1 \right\}^2}{2\psi_2^2} \right] \quad (14)$$

$$N_{m1} = \left( \frac{\bar{C}_1}{\sigma_1} \right)^{\beta_1} \quad (15)$$

$$N_{m2} = \left( \frac{\bar{C}_2}{\sigma_2} \right)^{\beta_2} \quad (16)$$

$$\bar{C}_1 = \left[ \frac{\bar{A}_1}{\sqrt{2}} \right] \left[ \frac{1}{\Gamma\left(\frac{2+\beta_1}{2}\right)} \right]^{1/\beta_1} \quad (17)$$

$$\bar{C}_2 = \left[ \frac{\bar{A}_2}{\sqrt{2}} \right] \left[ \frac{1}{\Gamma\left(\frac{2+\beta_2}{2}\right)} \right]^{1/\beta_2} \quad (18)$$

$$\lambda(N) = \frac{p(N)}{1 - F(N)} \quad (19)$$

$$\lambda(N) = \text{hazard rate}$$